

## Chapter

# 5

## Discounted Cash Flow Valuation

## Annuities and Perpetuities Defined

- Annuity – finite series of equal payments that occur at regular intervals
  - If the first payment occurs at the end of the period, it is called an ordinary annuity
  - If the first payment occurs at the beginning of the period, it is called an annuity due
- Perpetuity – infinite series of equal payments

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5.1

## Annuities and the Calculator

- You can use the PMT key on the calculator for the equal payment
- The sign convention still holds
- Ordinary annuity versus annuity due
  - You can switch your calculator between the two types by using the 2<sup>nd</sup> BGN 2<sup>nd</sup> Set on the TI BA-II Plus
  - If you see “BGN” or “Begin” in the display of your calculator, you have it set for an annuity due
  - Most problems are ordinary annuities

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## Annuity – Example 5.5

- You borrow money TODAY so you need to compute the present value.
  - 48 N; 1 I/Y; -632 PMT; PV = 23,999.54 (\$24,000)

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5.3

## Annuity – Sweepstakes Example

- Suppose you win the Publishers Clearinghouse \$10 million sweepstakes. The money is paid in equal annual installments of \$333,333.33 over 30 years. If the appropriate discount rate is 5%, how much is the sweepstakes actually worth today?
  - 30 N; 5 I/Y; 333,333.33 PMT; PV = 5,124,150.29

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## Buying a House

- You are ready to buy a house and you have \$20,000 for a down payment and closing costs. Closing costs are estimated to be 4% of the loan value. You have an annual salary of \$36,000 and the bank is willing to allow your monthly mortgage payment to be equal to 28% of your monthly income. The interest rate on the loan is 6% per year with monthly compounding (.5% per month) for a 30-year fixed rate loan. How much money will the bank loan you? How much can you offer for the house?

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## Buying a House - Continued

- Bank loan
  - Monthly income =  $36,000 / 12 = 3,000$
  - Maximum payment =  $.28(3,000) = 840$ 
    - $30 * 12 = 360$  N
    - .5 I/Y
    - 840 PMT
    - PV = 140,105
- Total Price
  - Closing costs =  $.04(140,105) = 5,604$
  - Down payment =  $20,000 - 5,604 = 14,396$
  - Total Price =  $140,105 + 14,396 = 154,501$

## Quick Quiz – Part 2

- You know the payment amount for a loan and you want to know how much was borrowed. Do you compute a present value or a future value?
- You want to receive 5000 per month in retirement. If you can earn .75% per month and you expect to need the income for 25 years, how much do you need to have in your account at retirement?

## Finding the Payment

- Suppose you want to borrow \$20,000 for a new car. You can borrow at 8% per year, compounded monthly ( $8/12 = .66667\%$  per month). If you take a 4 year loan, what is your monthly payment?
  - $4(12) = 48$  N; 20,000 PV; .66667 I/Y; PMT = 488.26

## Finding the Number of Payments – Example 5.6

- The sign convention matters!!!
  - 1.5 I/Y
  - 1000 PV
  - -20 PMT
  - N = 93.111 MONTHS = 7.75 years
- And this is only if you don't charge anything more on the card!

## Finding the Number of Payments – Another Example

- Suppose you borrow \$2000 at 5% and you are going to make annual payments of \$734.42. How long before you pay off the loan?
  - Sign convention matters!!!
  - 5 I/Y
  - 2000 PV
  - -734.42 PMT
  - N = 3 years

## Finding the Rate

- Suppose you borrow \$10,000 from your parents to buy a car. You agree to pay \$207.58 per month for 60 months. What is the monthly interest rate?
  - Sign convention matters!!!
  - 60 N
  - 10,000 PV
  - -207.58 PMT
  - I/Y = .75%

## Quick Quiz – Part 3

- You want to receive \$5000 per month for the next 5 years. How much would you need to deposit today if you can earn .75% per month?
- What monthly rate would you need to earn if you only have \$200,000 to deposit?
- Suppose you have \$200,000 to deposit and can earn .75% per month.
  - How many months could you receive the \$5000 payment?
  - How much could you receive every month for 5 years?

## Future Values for Annuities

- Suppose you begin saving for your retirement by depositing \$2000 per year in an IRA. If the interest rate is 7.5%, how much will you have in 40 years?
  - Remember the sign convention!!!
  - 40 N
  - 7.5 I/Y
  - -2000 PMT
  - $FV = 454,513.04$

## Annuity Due

- You are saving for a new house and you put \$10,000 per year in an account paying 8%. The first payment is made today. How much will you have at the end of 3 years?
  - 2<sup>nd</sup> BGN 2<sup>nd</sup> Set (you should see BGN in the display)
  - 3 N
  - -10,000 PMT
  - 8 I/Y
  - $FV = 35,061.12$
  - 2<sup>nd</sup> BGN 2<sup>nd</sup> Set (be sure to change it back to an ordinary annuity)

## Quick Quiz – Part 4

- You want to have \$1 million to use for retirement in 35 years. If you can earn 1% per month, how much do you need to deposit on a monthly basis if the first payment is made in one month?
- What if the first payment is made today?
- You are considering preferred stock that pays a quarterly dividend of \$1.50. If your desired return is 3% per quarter, how much would you be willing to pay?

## Effective Annual Rate (EAR)

- This is the actual rate paid (or received) after accounting for compounding that occurs during the year
- If you want to compare two alternative investments with different compounding periods you need to compute the EAR and use that for comparison.

## Annual Percentage Rate

- This is the annual rate that is quoted by law
- By definition  $APR = \text{period rate} \times \text{number of periods per year}$
- Consequently, to get the period rate we rearrange the APR equation:
  - $\text{Period rate} = APR / \text{number of periods per year}$
- You should NEVER divide the effective rate by the number of periods per year – it will NOT give you the period rate

## Computing APRs

- What is the APR if the monthly rate is .5%?
  - $.5(12) = 6\%$
- What is the APR if the semiannual rate is .5%?
  - $.5(2) = 1\%$
- What is the monthly rate if the APR is 12% with monthly compounding?
  - $12 / 12 = 1\%$
  - Can you divide the above APR by 2 to get the semiannual rate? NO!!! You need an APR based on semiannual compounding to find the semiannual rate.

## Things to Remember

- You ALWAYS need to make sure that the interest rate and the time period match.
  - If you are looking at annual periods, you need an annual rate.
  - If you are looking at monthly periods, you need a monthly rate.
- If you have an APR based on monthly compounding, you have to use monthly periods for lump sums, or adjust the interest rate appropriately if you have payments other than monthly

## Computing EARs - Example

- Suppose you can earn 1% per month on \$1 invested today.
  - What is the APR?  $1(12) = 12\%$
  - How much are you effectively earning?
    - $FV = 1(1.01)^{12} = 1.1268$
    - $Rate = (1.1268 - 1) / 1 = .1268 = 12.68\%$
- Suppose if you put it in another account, you earn 3% per quarter.
  - What is the APR?  $3(4) = 12\%$
  - How much are you effectively earning?
    - $FV = 1(1.03)^4 = 1.1255$
    - $Rate = (1.1255 - 1) / 1 = .1255 = 12.55\%$

## Computing Payments with APRs

- Suppose you want to buy a new computer system and the store is willing to sell it to allow you to make monthly payments. The entire computer system costs \$3500. The loan period is for 2 years and the interest rate is 16.9% with monthly compounding. What is your monthly payment?
  - $2(12) = 24 N$ ;  $16.9 / 12 = 1.408333333 I/Y$ ;  $3500 PV$ ;  $PMT = -172.88$

## Future Values with Monthly Compounding

- Suppose you deposit \$50 a month into an account that has an APR of 9%, based on monthly compounding. How much will you have in the account in 35 years?
  - $35(12) = 420 N$
  - $9 / 12 = .75 I/Y$
  - 50 PMT
  - $FV = 147,089.22$

## Present Value with Daily Compounding

- You need \$15,000 in 3 years for a new car. If you can deposit money into an account that pays an APR of 5.5% based on daily compounding, how much would you need to deposit?
  - $3(365) = 1095 N$
  - $5.5 / 365 = .015068493 I/Y$
  - 15,000 FV
  - $PV = -12,718.56$

## Quick Quiz – Part 5

- What is the definition of an APR?
- What is the effective annual rate?
- Which rate should you use to compare alternative investments or loans?
- Which rate do you need to use in the time value of money calculations?

## Pure Discount Loans – Example 5.11

- Treasury bills are excellent examples of pure discount loans. The principal amount is repaid at some future date, without any periodic interest payments.
- If a T-bill promises to repay \$10,000 in 12 months and the market interest rate is 7 percent, how much will the bill sell for in the market?
  - 1 N; 10,000 FV; 7 I/Y; PV = -9345.79

## Interest Only Loan - Example

- Consider a 5-year, interest only loan with a 7% interest rate. The principal amount is \$10,000. Interest is paid annually.
  - What would the stream of cash flows be?
    - Years 1 – 4: Interest payments of  $.07(10,000) = 700$
    - Year 5: Interest + principal = 10,700
- This cash flow stream is similar to the cash flows on corporate bonds and we will talk about them in greater detail later.

## Amortized Loan with Fixed Payment - Example

- Each payment covers the interest expense plus reduces principal
- Consider a 4 year loan with annual payments. The interest rate is 8% and the principal amount is \$5000.
  - What is the annual payment?
    - 4 N
    - 8 I/Y
    - 5000 PV
    - PMT = -1509.60

## Amortization Table for Example

Year	Beg. Balance	Total Payment	Interest Paid	Principal Paid	End. Balance
1	5,000.00	1509.60	400.00	1109.60	3890.40
2	3890.40	1509.60	311.23	1198.37	2692.03
3	2692.03	1509.60	215.36	1294.24	1397.79
4	1397.79	1509.60	111.82	1397.78	.01
Totals		6038.40	1038.41	4999.99	

## Quick Quiz – Part 6

- What is a pure discount loan? What is a good example of a pure discount loan?
- What is an interest only loan? What is a good example of an interest only loan?
- What is an amortized loan? What is a good example of an amortized loan?